Indexer – Summary Review of Computation

rev Daniel Brockman 20141208

Summary Review of Computation

Let us walk through our computations, using a hypothetical example of a Presentation Risk Index composed from two metrics, FICO and LTV. Specific numbers in this example are hypothetical. There may be better choices for the transformations, the shapes of the S-curves, and the choice of metrics.

Steps:

1. Transform each metric to a zero-to-one scale. Zero=low risk. One=high risk.
2. Apply S-curve to each transformed metric.
3. Compute distance measure.
4. Convert distance measure to a zero-to-one scale. Zero=low risk. One= high risk.

Tip: The 2-metric case has 4 points where the metrics assume extreme values:
NW = (low,high) = (0,1); index=(1-1/sqrt(2)) = .2929
SW = (low,low) = (0,0) = the point of lowest risk; index=(1-sqrt(2)/sqrt(2)) = 0
NE = (high,high) = (1,1) = SR = the point of supreme risk ; index=(1-0/sqrt(2)) = 1
SE = (high,low) = (1,0); index=(1-1/sqrt(2)) = .2929
The index you calculate should be very close to these theoretical values at these "corner" points. There are analogous "corners" for n metrics.

<u>Step 1: Transforming each metric:</u>

We use the formula

$$y(i) = (x(i) - M(lo)) / (M(hi) - M(lo))$$

where x(i) is the LTV for loan i, y(i) is the transformed metric for loan i, M(hi) is the highest-risk LTV we use (if LTV is greater, then we use M(hi)), and M(lo) is the lowest-risk LTV.

Examples…

We use two loans numbered 285 and 318, and we use four hypothetical loans NW, SW, NE and SE that have maximal and minimal values of LTV and FICO.

Suppose LTV is 48 for Loan 285. With M(hi)=200 and M(lo)=0, we calculate the transformed value yLTV = 0.24 = 48 / 200 = (48 – 0) / (200 – 0).

Suppose FICO is 655 for loan 285. With M(hi)=300, the highest-risk value, and M(lo)=850, we have yFICO = 0.35 = (-195) / (-550) = (655 – 850) / (300 – 850).

orig data			transformed metrics		extreme values		
Loan No.	**LTV**	**FICO**	**yLTV**	**yFICO**		**LTV**	**FICO**
NW	0	300	0.00	1.00	min	0	300
SW	0	850	0.00	0.00	max	200	850
NE	200	300	1.00	1.00			
SE	200	850	1.00	0.00			
285	48	655	0.24	0.35			
318	92	803	0.46	0.09			

Step 2: Applying the S-curve to each transformed metric:

In our example, for LTV, we chose shaping points

U:(xu,yu)=(0.5000.0.9500) and
V:(xv,yv)=(0.2500,0.0500).

xu=0.5, the transformed value when LTV=100, a high value which we want to correspond with a S-curve transformed LTV risk score of yu=0.95 (on a scale of 0.00 to 1.00). And xv=0.25, the transformed value of LTV=50, which we want to take the S-curve transformed LTV risk score of yv=0.05, a low risk by the LTV metric.

We calculate the u and v values for the LTV S-curve. Recall

$$Let\ u = -ln(yu) + ln(1 - yu)$$
$$Let\ v = -ln(yv) + ln(1 - yv)$$

So, u= -2.94444 = -ln(yu) + ln(1-yu)=-ln(0.95)+ln(1-0.95)=-(-0.051293)+(-2.99573),
and v= 2.94444 = -ln(yv) + ln(1-yv)=-ln(0.05)+ln(1-0.05)=-(-2.99573)+(-0.051293).

Then we use u and v to calculate a and b:

$$b = -(u - v) / (xu - xv)$$
$$a = -v - b*xv$$

b = 23.5555 = -(u-v)/(xu-xv)=-((-2.94444)-(2.94444))/((0.8)-(0.4))=-(-5.88888)/(0.4)
a = -8.83332 = -v-b*xv=-(2.94444)-(14.7222)*(0.4)=-2.94444-5.88888

Expressing our General Formula ($y = exp(a + b*x) / (1 + exp(a + b*x))$) for the S-curve in this special case,

$$SyLTV(yLTV) = exp(-8.83332 + 14.7222*yLTV) / (1 + exp(-8.83332 + 23.5555*yLTV))$$

Similarly, for FICO:

U:(xu,yu)=(0.4181.0.9500) and
V:(xv,yv)=(0.1091,0.0500).
u = -2.94444
v = 2.94444

b = 19.0579
a = -5.02365

*SyFICO(yFICO) = exp(-5.02365 + 19.0579*yFICO) / (1 + exp(-5.02365 + 19.0579*yFICO))*

orig data			transformed metrics		S-Curve	
Loan No.	**LTV**	**FICO**	**yLTV**	**yFICO**	**SyLTV**	**SyFICO**
NW	0	300	0.00	1.00	0.000	1.000
SW	0	850	0.00	0.00	0.000	0.007
NE	200	300	1.00	1.00	1.000	1.000
SE	200	850	1.00	0.00	1.000	0.007
285	48	655	0.24	0.35	0.040	0.850
318	92	803	0.46	0.09	0.881	0.032

Step 3: Compute the distance measure.

Recalling the initial "S" means the inputs were treated with the S-curve, we use the formula for the distance from SR, the point of supreme risk

$$Sd(i) = sqrt((SyLTV(i) - 1)^2 + (SyFICO(i) - 1)^2)$$

orig data			transformed metrics		S-Curve		Distance
Loan No.	**LTV**	**FICO**	**yLTV**	**yFICO**	**SyLTV**	**SyFICO**	**Sd**
NW	0	300	0.00	1.00	0.000	1.000	0.9999
SW	0	850	0.00	0.00	0.000	0.007	1.4095
NE	200	300	1.00	1.00	1.000	1.000	0.0000
SE	200	850	1.00	0.00	1.000	0.007	0.9935
285	48	655	0.24	0.35	0.040	0.850	0.9717
318	92	803	0.46	0.09	0.881	0.032	0.9748

Step 4: Convert distance measure to a zero-to-one scale. Zero=low risk. One= high risk.

Distance is the distance from SR, the point of supreme risk. A smaller distance is a larger risk. Point SW has all transformed metrics equal to zero (absence of risk), and so SW is the point most distant from SR. We use the formula to calculate the risk index

$$Syd(i) = 1 - Sd(i) / sqrt(2)$$

orig data			transformed metrics		S-Curve		distance	**Risk Index**
Loan No.	**LTV**	**FICO**	**yLTV**	**yFICO**	**SyLTV**	**SyFICO**	**Sd**	**Syd**
NW	0	300	0.00	1.00	0.000	1.000	0.9999	0.29
SW	0	850	0.00	0.00	0.000	0.007	1.4095	0.00
NE	200	300	1.00	1.00	1.000	1.000	0.0000	1.00
SE	200	850	1.00	0.00	1.000	0.007	0.9935	0.30
285	48	655	0.24	0.35	0.040	0.850	0.9717	0.31
318	92	803	0.46	0.09	0.881	0.032	0.9748	0.31

End.